CALCULATION OF THE INCOMPRESSIBLE LAMINAR BOUNDARY LAYER ON A PLATE WITH SLOT SUCTION

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Inzhenerno-Fizicheskii Zhurnal, Vol. 9, No. 4, pp. 433–437, 1965.

UDC 532.517.2

The dependence of the characteristics of an incompressible laminar boundary layer on the flow rate of fluid removed by suction through transverse slots on the surface of a plate is investigated on the basis of the momentum equation.

We shall evaluate the characteristics of the laminar boundary layer formed in the flow over a flat plate with transverse slots of an incompressible fluid at large values of the Reynolds number. By slots we understand porous transverse sections on the plate sursurface, in which the normal velocity component is constant and nonzero. We shall use a rectangular system of coordinates with the origin located at the plate leading edge, the x axis being along the plate surface, and the y axis normal to it. The momentum equation (the law of change of momentum) for an element of the boundary layer on the flat plate may be written as [1]

$$\frac{d\delta^{**}}{dx} + \frac{v_0}{U} = \frac{\tau_w}{\rho U^2}. \qquad (1)$$

Note that for a given flow velocity we assume the second term of (1) to be a constant in the region of a slot, and zero outside the slot.

Using (1), we first determine the characteristics of the laminar boundary layer for uniform (v_0 = const) suction over the entire plate surface.

In the new variables

$$\xi = (-v_0/U)^2 Ux/\nu \text{ and } t^{**} = -v_0\delta^{**}/\nu$$

equation (1) takes the form

$$t^{**}\frac{dt^{**}}{d\xi} - t^{**} = \zeta. \qquad (2)$$

It is known [1] that for arbitrary distribution of suction velocity along the porous plate, the dimensionless local friction coefficient is

$$\zeta = \zeta_0 - dt^{**}, \qquad (3)$$

where $\zeta_0 = 0.22$; d = 0.56 and $t^{**} \le 0$. Comparison of the results of calculations using (3) with data from numerical integration of the differential equations of the laminar boundary layer on analog computers has shown that for values of t^{**} in the range 0 to −0.5, the maximum error of (3) does not exceed 3% approximately [2].

We shall transform (2) using (3). After separating the variables and determining the limits of integration, we obtain the simple integral equation

$$\int_0^{t^{**}} \frac{t^{**}dt^{**}}{\zeta_0 + (1-d)t^{**}} = \int_0^{\xi} d\xi \qquad (4)$$

with the following boundary conditions: $t^{**} = t^{**}$ when $\xi = \xi$ and $t^{**} = 0$ when $\xi = 0$.

Equation (4) may be solved in quadratures:

$$\xi = \frac{t^{**}}{1-d} - \frac{\zeta_0}{(1-d)^2}\ln\left|1 + \frac{(1-d)}{\zeta_0}t^{**}\right|. \qquad (5)$$

The exact solution of the problem of determining the characteristics of the incompressible laminar boundary layer with uniform suction on a porous plate was obtained in [3] by numerical integration of the Prandtl differential equations.

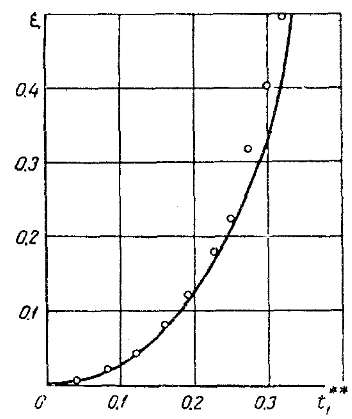

Fig. 1. Dependence of the universal ξ on the suction parameter t_1^{**} (the curve is the author's approximate solution, the points are the exact solution [3]).

Analysis of the data given in Fig. 1 shows satisfactory agreement to within several percent.

We shall solve the problem of calculating the characteristics of the laminar boundary layer on a plate with uniform suction (v_0 = const) beginning at distance L from the leading edge. Then, using (3), we shall reduce (1) to the following form:

$$\delta^{**}d\delta^{**}[\zeta_0\nu/U + (1-d)(-v_0)\delta^{**}/U]^{-1} = dx. \qquad (6)$$

To allow convenient comparison of the results of the approximate solution with the exact data obtained in [4], we introduce the following dimensionless values:

$$x_L = x/L;\ \delta_L^{**} = (\delta^{**}/L)\sqrt{UL/\nu};\ -v_{0L} = (-v_0/U)\sqrt{UL/\nu}.$$

We then reduce the differential equation (6) to the following form:

$$\delta_L^{**}d\delta_L^{**}/[\zeta_0 + (1-d)(-v_{0L})\delta_L^{**}] = dx_L,$$

and by integrating the left side from $\delta_L^{**} = 0.664$ to $\delta_L^{**} = \delta_L^{**}$, and the right side from $x_L = 1$ to $x_L = x_L$, we obtain

$$\frac{(\delta_L^{**}-0.664)}{(1-d)(-v_{0L})}-\frac{\zeta_0}{(1-d)^2(-v_{0L})^2}\times$$
$$\times\ln\left|\frac{1+(-v_0L)\,\delta_L^{**}(1-d)/\zeta_0}{1+(-v_0L)\,0.664\,(1-d)\,\zeta_0}\right|=\frac{x}{L}-1. \qquad (7)$$

Comparison of the given calculations based on the approximation (7) with the results of the exact Reynolds solution [4] also shows satisfactory agreement.

These comparisons have been made to illustrate the fact that the assumptions on which these calculations are based, and the approximate relations, give very satsifactory results in the sense of the permissible error of calculation.

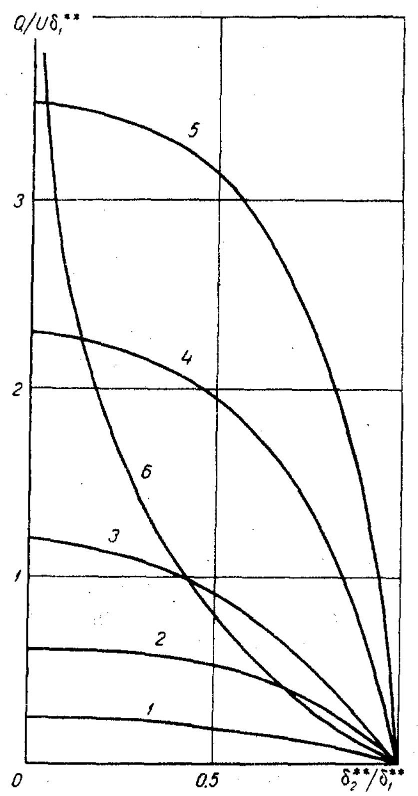

Fig. 2. Dependence of the dimensionless flow rate $Q/U\delta_1^{**}$ on the ratio of momentum thicknesses ahead of and behind the slots $\delta_2^{**}/\delta_1^{**}$, for various suction parameter values t_1^{**}: 1) $t_1^{**} = 0.1$; 2) 0.2; 3) 0.3; 4) 0.40; 5) 0.45; 6) from [5].

Without introducing new assumptions or approximations, we shall evaluate the basic relation between the characteristics of the laminar boundary layer and the flow rate $Q = -v_0 s$, when there is suction through a slot of width $s = x_2 - x_1$ located at distance x_1 from the plate leading edge. Here the following boundary conditions are satisfied when $x = x_1\delta^{**} = \delta_1^{**}$, and when $x = x_2\delta^{**} = \delta_2^{**}$. Integrating (6) in the range indicated, after some algebraic transformation we obtain the desired relation:

$$\frac{Q}{U\delta_1^{**}}=\frac{1}{(1-d)}\left(\frac{\delta_2^{**}}{\delta_1^{**}}-1\right)-\frac{\zeta_0}{(1-d)^2\,t_1^{**}}\times$$
$$\times\ln\left|\frac{1+t_1^{**}(1-d)\,\delta_2^{**}/\zeta_0\delta_1^{**}}{1+t_1^{**}(1-d)\,\zeta_0}\right|. \qquad (8)$$

It follows from an analysis of (8), that the dimensionless value of the mass flow rate required depends on the ratio of momentum thicknesses $\delta_2^{**}/\delta_1^{**}$ ahead of and behind the slot, and on the suction parameter t_1^{**} which is the Reynolds number based on the momentum thickness ahead of the slot and on the local suction velocity (Fig. 2).

In his calculations the author of [5] proceeded from geometric considerations and ignored the basic laws of hydrodynamics. It was assumed that for a given suction flow through the slot, the velocity profile behind the slot was formed by simply cutting off the lower part of the velocity profile ahead of the slot. As a result Lachmann incorrectly concluded that the dimensionless flow rate depends only on the momentum thickness ratio ahead of and behind the slot. This remark is of real significance, since Lachmann's relation has been made the basis of a unique method of practical calculation of the characteristics of the laminar boundary layer with suction through transverse slots located on the plate surface.

In this connection mention should be made of the results of [6], whose author made an unsound attempt to base the Lachmann relation on the momentum equations. The mistake that Colemann made in the calculations was to neglect the change of momentum due to friction forces in the slot region.

Without including any factual material, in his paper Colemann asserted that the Lachmann relation agreed satisfactorily with experimental data. From the viewpoint of the present study, this assertion would be valid only if Lachmann's theoretical data and the experimental material were compared for values of t_1^{**} and $\delta_2^{**}/\delta_1^{**}$ for which the Lachmann relation intersects the corresponding curves obtained in the present paper (Fig. 2). For example, for $\delta_2^{**}/\delta_1^{**} = 0.70$, 0.44, and 0.13, agreement with the experimental data may be satisfactory for values of the suction parameter $t_1^{**} = 0.2$, 0.3, and 0.4, respectively, since the Lachmann relation has proved to be valid only at particular values of the suction parameter.

Our recommendation is that approximate calculations of the characteristics be made using the relations given in Fig. 2, taking into account the value of the suction parameter. The recommended relation may be used approximately in calculations of the characteristics of the laminar boundary layer with slot suction from the boundary layer of wing profiles and bodies of revolution whose ratio of length to maximum width exceeds seven.

NOTATION

x) coordinate along plate surface; x_1 and x_2) coordinates of front and back edges of slot; s) slot width; y) coordinate normal to plate surface; U) free stream velocity; $v_0(x)$) distribution of normal velocity component over plate surface (local suction velocity); $\delta^{**}, \delta_1^{**}$ and δ_2^{**}) momentum thickness of boundary layer ahead of and behind slot; ν) kinematic viscosity of fluid; ρ) density of fluid; $t_1^{**} = -v_0\delta^{**}/\nu$) parameter describing suction from boundary layer; t_1^{**}) value of suction parameter at leading edge of slot; τ_w) local friction stress on plate surface; $\zeta = \tau_w\delta^{**}/\nu\rho U$) dimensionless local friction coefficient; $\xi = (-v_0/U)^2 Ux/\nu$) universal variable; Q) volume flow rate of liquid sucked through slot; $\zeta_0 = 0.22$, $d = 0.56$) constants.

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2 October 1964

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